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AN EXAMINATION OF THE USMC
COMBAT ACTIVE REPLACEMENT FACTOR
(CARF) DETERMINATION SYSTEM

by

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February, 1985

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Two means of estimating replacementfactor values for comparative purposes are examined. In one method, mean-time-to-loss estimates are employed in scenario oriented models. In the other, professional military judgement is employed as a means of assessing current values for combat active replacement factors.

AN EXAMINATION OF THE USMC COMBAT ACTIVE REPLACEMENT FACTOR (CARF) DETERMINATION SYSTEM

A Technical Report

February, 1985

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SUMMARY

Combat Active Replacement Factors, or CARFs, are logistics planning factors currently used by the U.S. Marine Corps as estimates of equipment losses in future conflicts. Adapted Army replacement factors are currently a prime source for CARF values, but verification of these values is difficult.

This report examines two alternate means of estimating CARF values for comparative purposes. The first employs mean-time-to-loss estimates for various equipment types, and several scenario-oriented models are given for mapping these values into CARFs.

Professional military judgement provides another way of estimating CARF values, and a procedure is given by which the judgement of many experts can be aggregated to provide CARF information. A demonstration of the procedure is given, employing actual data from twenty-three judges who rated "chance of loss" for twenty-one equipment types.

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I. INTRODUCTION

Combat Active Replacement Factors, or CARFs, are logistics planning factors currently used by the U.S. Marine Corps. As estimates of equipment losses in future conflicts, their values have a significant impact on procurement, stockpiling, and plans for shipping requirements.

Concern over the accuracy of CARF values led to the establishment by the Marine Corps in 1983 of a standing board for the review of materiel replacement factors. The board's specified functions include the review of " . . . replacement factors for new materiel entering the Marine Corps Supply System, existing replacement factors for materiel presently in the Marine Corps Supply System, and other services' replacement factors . . . where such factors would prove useful in confirming the validity of common item factors."[1] The board's task is a difficult one: the current Marine Corps primary source of CARF values is that of adapting Army replacement factors for like items, but the Marine Corps lacks available war gaming and simulation resources to permit careful verification of replacement factors by these means. This report discusses some ways of generating comparative CARF values (for verification use) using procedures which incorporate military judgement and do not require computerized war games.

Combat Active Replacement Factors (CARFs)

The War Reserve Policy Manual of the Marine Corps states that a replacement factor is ". . . the estimated percentage of equipment in use that will require replacement during a given period due to wear-out beyond repair, enemy action, abandonment, pilferage, and other causes, except catastrophies. . . . The Marine Corps expresses replacement factors as quantities required for a 30-day period." Further, "The combat active replacement factor will be applied for units during those periods when they are actually in active combat operations. A force in contact with the enemy is considered to be active combat. The combat active replacement factor reflects anticipated combat attrition of equipment, on a 30-day basis, incident to amphibious operations and other combat operations normal to the FMF".[2] Numerical values for CARFs for a wide variety of equipment items are listed in the Marine Corps Table of Authorized Materiel.[3]

With Marine Corps support, several studies were conducted in the early 1980s in an effort to develop a procedure to generate CARF values.[4,5] These led to a decision to use adapted Army replacement factor values.[6] In a related effort a CARF Determination System Users Manual was prepared.[7] Concern over verification has remained, and in 1985 a number of existing war games were surveyed with regard to their usefulness to the Marine Corps for CARF generation and verification.[8]

This Report

This report looks at several ways of employing military judgement to estimate CARFs for verification purposes. Although CARF values could be subjectively estimated directly, one alternative is to employ estimates of the average combat survival time of an item, and in the next section we will look at ways in which Mean Time Until Loss (MTTL) values may be converted into CARF values.

Another way to acquire professional military judgement about CARF values is to ask that various equipment items simply be ranked with regard to their CARF values. Section III of this report explains how such rankings by many judges may be combined to yield CARF values for the various items. A demonstration of this method is then given, scaling chance of loss for 21 functional areas of equipment. The report concludes with appendices providing some cautions regarding CARF generation and use, and the mathematical derivation of the scaling method which uses rankings.

III. CARF GENERATION AND VERIFICATION USING MTTL ESTIMATES

There are alternatives to the use of computerized war games and simulation to obtain values for CARFs. It is possible to produce estimates directly, perhaps using professional military judgement and experience. In some cases CARF values so generated may be preferred to those obtained from combat models, since there may be more clarity about what the number was based upon, and what considerations went into its estimation.

A variation on estimating CARFs directly is to use professional judgement to estimate a related measure, the mean time until loss, or MTTL. The MTTL is the average time one would expect an item to survive in the combat environment (the analogy to MTBF is appropriate). For some items and some individuals asked for judgements, the MTTL may be a more natural value to estimate directly than a replacement factor.

Given an estimate of the item's MTTL, however, the relationship between MTTL and CARF is not direct. What is needed are procedures by which an MTTL estimate for an item could be used to produce a CARF value for the item. Such procedures necessarily depend upon the process by which the losses occur. We shall look at several kinds of scenarios by which losses could occur in combat, and then develop relationships permitting a CARF value to be obtained from an MTTL value. These loss processes include, for a specific item:

1. Cases where all items of that type are vulnerable (subject to possible loss) initially and throughout the combat period, at the same rate.

2. Cases where subsets of the items in use have different loss rates.

3. Cases where the rate of loss changes at some designated time during the combat period. This permits representation of an amphibious phase, or a force buildup.

4. Cases where one item is vulnerable initially and throughout the combat period at the same rate, but its replacements are not vulnerable until put into use.

Development of MTTL-CARF relationships for these loss processes involves characterization of the probabilistic factors by which the loss of an item occurs.

Loss Process Characteristics

We will look at a loss process where the event that an item is lost is independent of the loss of any other on-line item of the same kind, and where the chance of a surviving item being lost on the next day is independent of how many days it has already been in combat. This means that the individual item's loss rate may be considered constant over the period of time we are examining.

For constant loss rates and independent losses, the probability distribution for the time an item survives in combat will be the exponential distribution.[9] Let t be the combat survival time for a specific item, and let R be that item's loss rate. (The units of R are items per day.) The density function for the time t until the item is lost is

$$f(t) = Re^{-Rt} \quad , \quad t>0, \quad R\geq 0 \ .$$

From this we may immediately obtain the probability that the item is lost on or before time T, F(T), as

$$F(T) = Pr(t\leq T) = 1 - e^{-TR} \quad .$$

We also have 1 - F(T), the probability that the item is still surviving just after time T, as

$$1 - F(T) = Pr(t>T) = e^{-TR} \quad .$$

Since the expected value or average value of an exponentially distributed random variable is the reciprocal of its parameter, we now have the mean time until loss MTTL as

$$MTTL = 1/R \quad .$$

Accordingly, the chance that the item is not lost during 30 days of combat is

$$Pr(\text{Not lost during first 30 days}) = e^{-30/MTTL},$$

or more generally,

$$Pr(\text{Not lost during first D days}) = e^{-D/MTTL}. \qquad (1)$$

With this characterization of the general loss process established, we can now look at various loss scenarios.

<u>CARFS When All Items are Vulnerable Initially with the Same Loss Rate for 30 Days</u>

We look first at the case where all items are committed and vulnerable at the same loss rate throughout the 30-day period. The chance that an item is lost during the 30 days is

$$1 - e^{-30/MTTL} \quad .$$

If n items are the initial in-use amount, committed with independent losses, then the number of items x that would be lost during the 30-day period will be binomially distributed, and we have

$$Pr\begin{pmatrix}\text{x out of n} \\ \text{lost in 30 days}\end{pmatrix} = \frac{n!}{x!(n-x)!}(1-e^{-30/MTTL})^{x}(e^{-30/MTTL})^{n-x},$$

$$x = 0,1, \ldots, n \ . \qquad (2)$$

The average number (out of n) lost in 30 days is simply the mean of the binomial distribution, or

$$\text{Ave number lost} = n(1 - e^{-30/MTTL}) \quad .$$

From this we may readily obtain a CARF value:

$$\text{CARF} = \frac{\text{(Ave number lost in 30 days)(100)}}{\text{(in-use amount)}},$$

or

$$\boxed{\text{CARF} = (1-e^{-30/MTTL})(100)} \ . \qquad (3)$$

Equation (3) permits a CARF to be computed from an MTTL estimate for the case where each item in the in-use amount is committed initially and at the same loss rate, and where losses are independent.

Another use of (3) in the verification of an existing CARF value would ask if the existing CARF value would yield an MTTL that seemed reasonable. Here, we could use

$$MTTL = -30/\ln(1-CARF/100) \quad . \tag{4}$$

The association between CARF and MTTL for this case is illustrated numerically by the values in Table 1.

TABLE 1. Some Associated CARF and MTTL Values for the Case of All Items Vulnerable Initially, and at the Same Loss Rate for 30 Days.

MTTL: Ave. Number of Days Before Loss	CARF, %
20 days	78 %
30	63
40	53
50	45
100	26
200	14
350	8
500	6

CARFs Where Not All Items Have the Same Loss Rate

If all items are initqlly vulnerable, but with different (constant) loss rates for the 30 day period, then CARF generation from the MTTL values is an extension of the case where all items have the same loss rate. For a general formula, let proportions $p_1, p_2, \ldots, p_k$ of the in-use amount n have mean times until loss $MTTL_1$, $MTTL_2$, ... , $MMML_k$. The average number lost in 30 days for any subgroup i is

$$p_i n(1 - e^{-30/MTTL_i}) \quad ,$$

and we have

$$\boxed{CARF = \left(\sum_{i=1}^{k} p_i (1 - e^{-30/MTTL_i}) \right) (100)} \quad . \qquad (5)$$

As an example of the use of (5), suppose half of the in-use amount have MTTL of 200 days, one-quarter have MTTL of 100 days, and the remaining quarter have MTTL of 30 days. Then from (5),

$$CARF = 100(\tfrac{1}{2}(1 - e^{-30/200}) + \tfrac{1}{4}(1 - e^{-30/100}) + \tfrac{1}{4}(1 - e^{-30/30}))$$

$$= 100(\tfrac{1}{2}(0.14) + \tfrac{1}{4}(0.26) + \tfrac{1}{4}(0.63)) \;=\; \underline{29.25\ \%} \quad .$$

This is, of course, simply a weighted average of CARF values from Table 1.

CARFs Where All Items are Vulnerable Initially and There is a Change in the Loss Rate During the 30-Day Period

It may sometimes be of interest to construct a CARF for a situation where there is a change in the combat scenario during the 30-day period. One immediate example of this is the case where the first portion of the 30 days is an amphibious operation, and the loss rate might subsequently change. For this type of situation, we let $1/MTTL_1$ be the loss rate for the first D days where x_1 items are lost, and $1/MTTL_2$ be the loss rate for the remaining (30 - D) days where x_2 items are lost. Here, total losses for the 30-day period are $x_1 + x_2$, but the problem is complicated by the fact that the initial in-use amount for the second period is $n - x_1$, i.e., it depends upon losses during the first period.

In particular, we have the average total losses for the 30-day period as

$$E\left[x_1+x_2\right] = E\left[x_1\right] + \sum_{x_1=0}^{n} E\left[x_2 \middle| x_1\right] \Pr(x_1) \quad . \qquad (6)$$

Substituting (2) and (3) into (6) yields

$$E\left[x_1+x_2\right] = E\left[x_1\right] + \sum_{x_1=0}^{n} (n-x_1)\left(1-e^{-(30-D)/MTTL_2}\right) \cdot \left(\frac{n!}{x_1!(n-x_1)!}(1-e^{-D/MTTL_1})^{x_1}(e^{-D/MTTL_1})^{n-x_1}\right)$$

$$= n(1-e^{-D/MTTL_1}) + (1-e^{-(30-D)/MTTL_2})(n - n(1-e^{-D/MTTL_1})) .$$

This simply gives us

$$E\left[x_1+x_2\right] = n\left(1 - e^{-(D/MTTL_1 + (30-D)/MTTL_2)}\right).$$

The CARF would be

$$\boxed{CARF = \left(1 - e^{-(D/MTTL_1 + (30-D)/MTTL_2)}\right)\left(100\right).} \quad (7)$$

More Complex Scenarios

The methods of the two preceding sections may be applied for the purpose of generating CARFS in other, more complex scenarios. A single example would be that of a reserve force committed at some time during the 30 days. The total initial number of items might be divided between an on-line force and a reserve force, with different MTTL values. Commitment of the reserve at some time D during the 30-day period would decrease its MTTL, and possibly increase the MTTL of the initial on-line force. Equations developed in the preceding sections offer the tools to construct CARFs for this scenario, and for others with more complexity.

CARFS Where an On-line Item is Replaced by a Heretofore Invulnerable Item

The expressions for generating CARFs in the previous sections have all been based upon a situation where all items were initially vulnerable (although loss rates could have been zero). A different loss process which we now examine would place one item on line, and structure its (possibly repeated) replacement with heretofore invulnerable items.

If the supply of reserve items is very large, or large enough so that the chance of its exhaustion is negligible, we can structure this case by noting that the loss process is simply a sequence of exponentially distributed time intervals over a 30 day period. If we ignore boundary conditions, the number of such intervals would be a Poisson distributed random variable with a mean of 30/MTTL.[9] The CARF would be

$$\boxed{\text{CARF} = \left(\frac{30/\text{MTTL}}{\text{in-use amount}}\right)\left(100\right)} \quad . \tag{8}$$

For CARF verification we could ask if

$$\text{MTTL} = \frac{(30)(100)}{(\text{CARF})(\text{in-use amount})} \tag{9}$$

was a reasonable number.

Equations (8) and (9) may be a reasonable approximation for an item with a low CARF value. If the chance of running out of replacements is not negligible, however, then (8) will yield an overstated CARF value. We can correct for this (at the cost of simplicity) as follows. Let n be the initial amount of an item, and x be the losses in 30 days. Then, with the Poisson probability distribution we can write

$$f(x) = \begin{cases} \dfrac{(30/\mathrm{MTTL})^x e^{-30/\mathrm{MTTL}}}{x!} & , \; x=0,1,\ldots,n-1 \\[2ex] \displaystyle\sum_{x=n}^{\infty} \dfrac{(30/\mathrm{MTTL})^x e^{-30/\mathrm{MTTL}}}{x!} & , \quad x = n \end{cases} \tag{9}$$

as the loss distribution. Taking expected values, this yields a CARF expression

$$\boxed{\mathrm{CARF} = \left(\frac{100}{n}\right)\left(\sum_{x=0}^{n-1} \frac{x(30/\mathrm{MTTL})^x e^{-30/\mathrm{MTTL}}}{x!} + n\left(1 - \sum_{x=0}^{n-1} \frac{(30/\mathrm{MTTL})^x e^{-30/\mathrm{MTTL}}}{x!}\right)\right)} \tag{10}$$

which is easily programmed for numerical solutions.

Remarks

The methods given in this section provide a way of generating CARF values without resort to war games or to simulation. Basic to this method is the estimation of values for MTTL, the mean time until a loss, and the method will be most applicable to situations where the MTTL can be estimated with more confidence than the needed CARF can be estimated.

The process assumptions leading to the use of the exponential distribution for time until loss are analogous to the constant failure rate assumption which is so widely used in reliability theory.[10] In the reliability context this assumption is widely discussed in that it portrays chance failures, but not those stemming from increased age of the item, or wearout. Constant failure rates, however, are widely used and are essentially mandated by DOD needs for MTBF, rather than mission, reliability specifications. For many items in our 30-day combat scenario, the effects of longevity on item loss may be negligible, and the constant loss rate structure as we have employed it not a source of excessive concern.

In the next section we will look in more detail at a procedure by which professional judgement could be used collectively to estimate or verify CARFs or MTTLs.

III. CARF GENERATION AND VERIFICATION USING MILITARY JUDGEMENT

A frequent procedure for eliciting expert opinion from knowledgeable individuals is that of asking them to do some form of ordinal rating of various items with regard to some property. Judges usually can provide ordinal ratings or ranking with much greater confidence than they can state numerical values. For example, from professional expertise one should be able to state which of two items should have the greater CARF value, and do this with more assurance than to state estimated CARF values for each of the two items.

The method given on the following pages allows ordinal information furnished by judges to be combined with a model of judge behavior to obtain an interval scale. Thus while judges may be asked only to rank items in terms, say, of their loss rates, the collective inputs of many judges permit and interval scale to be inferred without arbitrary scoring.[11] Then, specification of CARF values for two of the items should yield the needed ratio-scaled CARF estimates for all the items being considered.

Although ordinal ratings by judges may arise in many diverse situations, in too may cases such data is gathered without a clear notion of how it will be processed in order to have a resulting scale that collectively represents the inputs of the judges.

A commonly used (but not recommended) approach to dealing with a set of lists of rankings is to award arbitrarily chosen numerical values for various ranking positions, and then to compute an average score for each item. For example, if there are ten items to be scaled, one might award nine points if a judge ranks an item first, eight points if he ranks it second, and so on. The procedure imposes an interval scale interpretation upon the ordinal data furnished by the judges, and in most cases is very difficult to justify. Further, from a sensitivity analysis point of view, it may be shown that the results, and even their rank order, often depend dramatically upon the scoring system used.[12]

On the following pages we will first talk about various ways to obtain expert judgements from knowledgeable persons. Then, we will present a step-by-step procedure to convert the rankings to an interval, and then to a ratio scale permitting CARF estimation. The mathematics supporting the procedure are presented in Appendix B.

Obtaining Information From Judges

There are several different ways of obtaining rankings from judges, and when selecting a particular approach one would want to consider both the number of items for which values are sought, and the number of judges who are to be queried. Tradeoffs may have to be made between the effort that will be required of a judge and the amount of confidence one wishes to have in the resulting scale values.

We begin by discussing various ways in which judges might record a full ranking of the items. Then, we will describe a paired comparisons approach used by psychologists and others.

Ranking all the Items

A direct scheme for querying a judge is to present him with a set of items together with instructions that he is to rank them in terms of their CARF values. (Alternatively, we could ask him to do the ranking in terms of MTTL, or loss rate, or chance of loss.) As part of these instructions, he should be encouraged to rearrange and adjust his ordering until he is satisfied with it.

One way for the judge to express his rankings is to write the names of the items in a ranked list. The list gives him and others an easily inspected display of his work, and thereby facilitates any subsequent rearrangement he may wish to do. Rather than writing the names of the items, the judge could write symbols (numbers or letters) representing the items. (For example, he would place item Number 4 first, item Number 1 second, and so on.) This is easy to do, but his opportunity for easy inspection is hindered by the coding. Further, unless instructions are very clear, there can be ambiguity about the results in that if a 3 is listed first, he might have meant that item Number 1 was third, rather than item Number 3 first. It is generally felt better to ask the judge to use letter symbols or abbreviations of the item names. Also, it is essential to make clear which goes at the top of the list: the item with the highest chance of loss, or the item with the lowest chance of loss.

A different procedure asks the judge to work directly with the given list of items, indicating on that list which is ranked first, second, and so on. This approach makes it very difficult for the judge to inspect his work to see if he is satisfied with the rankings he has given.

A very efficient scheme tending to encourage rearrangement and to eliminate logic errors is to have the items written on cards - - judges spread out the cards and arrange them to their satisfaction.

Occasionally, a judge will feel that he cannot discriminate between two items in terms of their chance of loss. Also, there will be cases where a judge wishes not to rank an item because he feels he does not know enough about it. Although one will wish to encourage judges to rank all items to the extent that they are able, it is undesirable to require them to furnish information which they do not feel they can express. In the method given in this report, information from judges who omit items or who list ties may be used completely.

The Method of Paired Comparisons

A method of obtaining ordinal information from judges which has received considerable attention in the literature of Psychology is the Method of Paired Comparisons.[13,14,15,16] Here a judge is presented with pairs of items and is asked to indicate (mark, circle, etc.) which item in the pair has the higher loss rate. This data may be incorporated directly into the scaling procedure that will be presented here. If there are n items to be rated, this involves $n(n-1)/2$ comparisons, so that if there are 20 items, the judge would look at 190 pairs. Aggregate rankings, which are inherent in the total ranking approach, will not be possible. Ties are treated as omitted comparisons.

Apart from the large number of comparisons that may have to be made, another problem with paired comparisons is that of insuring logical consistency. A judge might rank A over B, B over C, and then C over A. It may be argued that such an inconsistency is properly part of the judge's expression of his feelings, and should be accepted. The practical problem that remains is the judge's efforts to try to avoid this, even if asked not to. This adds to the busywork of his task, and may influence the spontaneity of his ratings.

Procedure for Obtaining Scale Values

1. Data from the judges' ratings (ordinal) may be tallied in an f_{ij} array, where f_{ij} is the number of judges who ranked item j as possessing a greater, say, loss rate than item i. When this is done, $f_{ab} + f_{ba}$ will be the number of judges who compared item a with item b.

2. The p_{ij} array may be computed from the f_{ij} array as the proportion of judges comparing i and j who rated item j as having a greater loss rate than item i. Here,

$$p_{ij} = \frac{f_{ij}}{f_{ij} + f_{ji}} \quad . \tag{11}$$

Note that $p_{ab} + p_{ba} = 1$. On the diagonal of the p_{ij} array, we set $p_{ii} = 0.5$

3. Using a table of the standard normal distribution F(z), we prepare an array z_{ij} where z_{ij} is the standard normal value corresponding to p_{ij}, or

$$p_{ij} = \int_{-\infty}^{z_{ij}} f(z)dz = F(z_{ij}) \quad .$$

We omit (leave blank) z_{ij} entries corresponding to $p_{ij} > 0.98$, or $p_{ij} < 0.02$. The z_{ij} array will have zeros on the main diagonal, and if a and b are items, $z_{ab} = - z_{ba}$.

4. If the z_{ij} array has no missing values, we use the column averages as the preliminary scale values for the n items. These column averages

$$S_j = \frac{\sum_{i=1}^{n} z_{ij}}{n} \quad , \; j=1,2,\ldots,n \; . \qquad (12)$$

are the loss rate values for the n items on an interval scale, that is, a scale with arbitrary origin and unit.

5. If the z_{ij} array has missing values, the procedure is as follows. We compute column averages for those columns which are complete, and use these column averages as scale values for those items. For incomplete columns, we write a set of linear equations of the form

$$n_j S_j - \sum_{i \in \phi_j} S_i = \sum_{i \in \phi_j} z_{ij} \; , \qquad (13)$$

where ϕ_j denotes the set of n_j elements in column j of the z_{ij} array. Substituting the scale values already determined by averaging values in the complete columns, we solve the set of simultaneous equations to obtain the remaining scale values.

6. The preliminary scale values S_j which result from the foregoing are on an interval scale, and may be changed to other interval scales by changing origin and unit via any linear transformation of the form

$$S_j' = A + BS_j, \quad B>0 \quad . \tag{14}$$

To obtain the final scale values we need to add information which will specify the proper unit and origin. This can be done by estimating final scale values for exactly two of the items, call them g and h. (For example, we might say that the loss rate for item g is $L_g = 10$, and the loss rate for item h is $L_h = 3$.) Then, the final scale L_j is obtained by transforming all the preliminary scale values S_j by

$$L_j = \left(\frac{S_g L_h - S_h L_g}{S_g - S_h} \right) + \left(\frac{L_g - L_h}{S_g - S_h} \right) S_j \quad . \tag{15}$$

The values found by (15) will be the end result of the judges ranking of the items. If items were ranked in terms of their CARF values, then after estimating CARF values for two items g and h, Equation (15) yields the estimated CARF values for all of the items. One useful way to employ this method is to add to the list of items which are to be scaled to obtain CARF values two additional items for which CARF values exist and are felt to be satisfactory. Judges rank all the items, and at the end of the data analysis the two items with known CARFs are used to position the scale.

Remarks

The procedure presented here employs collective military judgement to generate estimates of factors describing loss properties of various types of equipment. Input data is simply in the form of ranked lists, while the final values are essentially ratio scaled when the procedure is used to estimate such factors as loss rates, MTTL values, or CARFs.

The method has the disadvantage that it is based upon disagreement among judges; if judges are in close agreement in their ranked lists, many items will be unscaled except in an ordinal sense. A related concern is that a reasonably large number of judges is needed. One cannot, for example, expect much success if only nine or ten judges are available.

A demonstration of this procedure employing real judges and data is given in the next section, where we look at obtaining values for <u>chance of loss</u> for 21 functional areas of equipment.

IV. DEMONSTRATION OF THE METHOD: CHANCE OF LOSS FOR TWENTY-ONE FUNCTIONAL AREAS

As a demonstration of the use of military judgement to estimate chance of loss of various kinds of equipment, twenty-three knowledgeable judges were asked to rank twenty-one types of equipment in terms of "chance of loss of individual items". The equipment types were identified by management function area codes, and the list of items is given in Table 2.

Raw Data

A replica of the ranking form is shown in Table 3. Judges wrote area code numbers on their lists, rather than equipment names. Of the twenty-three judges, only five rated all the items: most omitted two or three, although one ranked only five items.

The instructions said nothing about writing ties on the same line, but seven of the judges did this. Since the scaling procedure handles ties and omitted items, all the data furnished by the judges was used.

It is a characteristic of the scaling method that if all judges rank an item first (or all rank it last), then that item cannot be scaled except in an ordinal sense.

TABLE 2. Table of Management
Functional Area Codes

FA Code	Description
10	Radios
13	Air Command/Control Equipment
14	Air Support Radar/IFF Equipment
16	Electronic Equipment
17	Ground Support Radar
19	Intelligence/Surveillance Equipment
20	Generators
21	Environmental Control Equipment
23	Earth Moving Equipment
26	Materials Handling Equipment
29	Engineer Support
30	Trucks
35	Towed Motor Transport Equipment
40	Tanks
41	Amphibious Assault Vehicles
42	Light Armored Vehicles
43	Artillery
46	Infantry Weapons
49	Missile Systems
50	High Density/Low Deadline
11	Communications Support Equipment

TABLE 3. Demonstration Questionaire

Please rank the listed functional areas in terms of the chance of loss of individual items in the functional area. For example, if you think that Towed Motor Transport Equipment has the highest chance of loss, you would write the name, or the code '35' on the top line. Omit any functional areas you are unable to rank.

HIGHEST CHANCE OF LOSS ____________

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LOWEST CHANCE OF LOSS ____________

This was the case with Item 50, High Density/Low Density. It was ranked by fewer judges (nine) than any other item, and most placed it last on their lists. No two judges ranked it above any other specific item. As a result, it was dropped from the analysis, to reappear in the final scale where it would be declared as having a lower chance of loss than the lowest item scaled.

Of the remaining twenty items, judge's omissions were scattered over the items. The exception was Item 11, Communication Support Equipment, which was ranked by only ten of the twenty-three judges. Since frequencies were good on the other items, Item 11 was allowed to remain in the analysis.

Obtaining the Interval Scale

Scaling twenty items simultaneously is a larger than typical problem, and it was necessary to extract paired information from the lists for the f_{ij} array by using a computer, since the task was too large to be done by manual methods. When cells with p_{ij} values which were too large or too small were deleted, there remained six complete columns out of the original twenty. This meant that six items could have their scale values computed easily, and then a set of 14 simultaneous linear equations would have to be solved to obtain the remaining scale values.

An easier approach was taken. The data was divided into two scaling problems, each with eleven items so that there were two in common. The sets of items were:

#1: 10,13,14,16,17,19,20,40,41,42,43; and

#2: 10,11,16,21,23,26,29,30,35,46,49.

Scales for the items in each problem were found using the procedures described in Section III of this report, and then the two scales were merged by a linear transformation based upon the two common items.

This resulted in an interval scale for chance of loss for the 20 equipment items; the scale is shown in Figure 1. Note that no numbers appear on the scale since they would be misleading: scale unit and origin are, at this point, totally arbitrary.

Reference Points

Two of the items were assigned chance of loss values: Trucks at 0.0480, and Tanks at 0.3902. These values were specified independently of the ranking information. Interval scale values for the entire set of equipment areas as shown in Table 4 were transformed to a scale that would include the two reference points. This was done via Equation (15), which in this case was

$$L_j = -0.4658 + 0.01S_j \quad .$$

FIGURE 1. Interval Scale for Chance of Loss Data

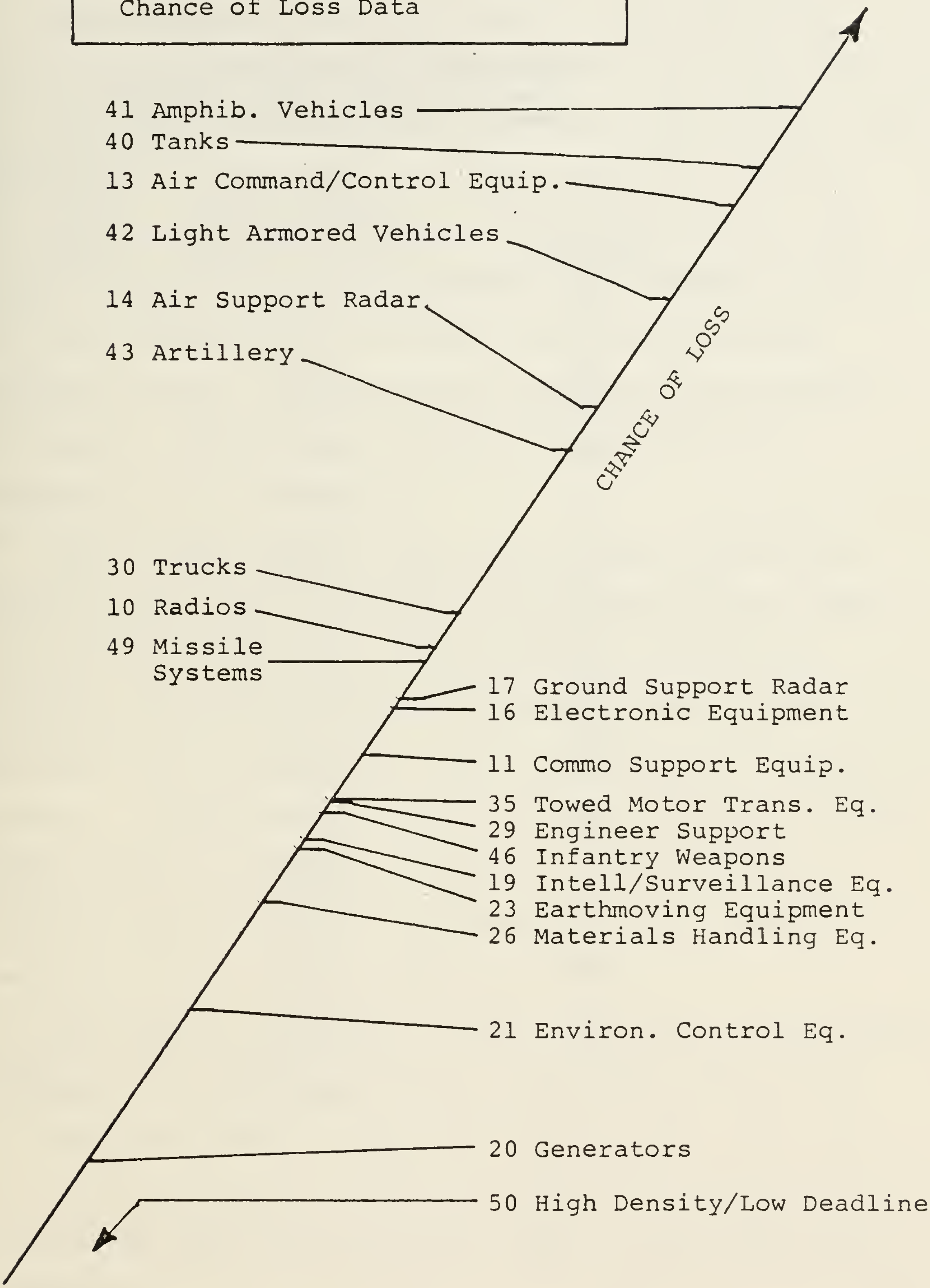

TABLE 4. Scaling Results for Twenty-one Equipment Functional Areas, Chance of Loss

FA Code	Interval Scale High 90, Low 10	Chance of Loss, based upon: $L_{30} = 0.0480$ $L_{40} = 0.3902$
10	48.9	0.0221
11	41.1	-0.0558
13	82.6	0.3583
14	67.1	0.2036
16	44.1	-0.0258
17	44.5	-0.0218
19	34.6	-0.1206
20	10.0	-0.3660
21	21.9	-0.2473
23	33.9	-0.1276
26	29.3	-0.1735
29	37.3	-0.0936
30	51.2	0.0480*
35	37.2	-0.0947
40	85.8	0.3902*
41	90.0	0.4321
42	75.5	0.2874
43	63.7	0.1697
46	36.9	-0.0976
49	47.1	0.0041
50	Below 10	Below -0.366

The resulting values are shown in Table 4.

The presence of negative values in the right-hand column of Table 4 deserves comment, since the terms "chance of loss" could be thought of a denoting a probability, and thus a non-negative value. In fact, there is nothing in this method to restrict values from being negative. Had the scale results all been positive, they probably would have been interpreted as probabilities without thought of justification. The presence of negative values serves as a useful reminder that "chance of loss" need not necessarily be a probability.

On the other hand, if one insists that probability values should result, then one can ask where errors might lie. The basic question when attempting to confirm a value by generating it from a different source is what to do when the two values differ. (Agreement supports the existing value.) In this case, if we insist that the chance of loss values should lie between zero and 1.0, then judge and reference values are in disagreement: one, the other, or both are in error to a conspicuous extent.

We note that the judges rated 14 of the 21 items (more than half) as having a chance of loss lower than Trucks, and that the reference value for truck, 0.048, was, as a probability, already a low value. Had the reference value for Trucks been higher, positive scale values would have resulted.

One way to investigate this would be to repeat the study, possibly with a fewer number of items, but retaining #30, Trucks, and #40, Tanks. Then,to the list of items that judges would be asked to rank we would add two, described as "A functional area whose chance of loss is 0.048" and "A functional area whose chance of loss is 0.39". If these values are consistent with military judgement, about half the judges would rank trucks above 0.048, and about half below. Similarly for tanks, about half would rank tanks above 0.39, and about half below.

Finally

This demonstration served as a first effort to use collective rankings to value factors such as chance of loss. The data provided a judgement-based interval scale for twenty types of equipment, and disagreed in a ratio scale sense with two reference points.

It is hoped that the work presented here will be useful to those interested in applying judgemental methods to the verification of CARFs and similar measures.

APPENDIX A

SOME CAUTIONS REGARDING CARF GENERATION AND USE

There are many ways to estimate a CARF value, but in essence the estimate of losses upon which the CARF is based will be in one of two basic forms. Either the entire <u>loss ratio</u> will be estimated (as a percentage, proportion, or a fraction), or the <u>number of items</u> that will be lost will be estimated.

In the case where the loss ratio is estimated, the denominator of that ratio is left implied. An example would be an estimate using military judgement, "I think we'd lose 10% of that kind of equipment". Here it is not clear what quantity the 10% refers to. It is tempting to assume that the reference is to the T/E allowance for forces in contact with the enemy, consistent with CARF definition and interpretion. A person providing the estimate who has worked with values reflecting the quantity of item in the FMF, however, might be thinking of that reference when he states 10%, while a company commander might be using a different reference. Generally, when estimates are given directly as loss ratios, there is no guarantee that the reference will be the desired one unless this is made very clear to the individual making the estimate. Errors can be substantial. Also, this kind of estimate probably requires knowledge of the in-use amount.

An alternative form of loss estimate upon which a CARF might be based is the number of items lost and requiring replacement in a 30-day period. This is the form of estimate that would result when a computerized combat model is used to produce a CARF value. A direct statement of losses could also come from military judgement, e.g., "I think we'd lose two". Here the stated estimate is independent of the reference value, at least explicitly, and thus the denominator for the CARF is left to be specified separately.

Figure 2 shows the anatomy of a CARF, detailing the components involved in producing a number which satisfies the definition of a replacement factor,viz., the expected percentage of equipment that will require replacement . . . for a 30-day period. Our interest here is not only the numerator and denominator, but also the fact that the definition requires a CARF to be a percentage. Real and potential difficulties with both of these characteristics of a CARF are identified and discussed in this appendix.

<u>The Denominator: The "In-Use" Amount</u>

A CARF is a percentage, and as such is essentially dimensionless. When used for planning and other purposes,

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The CARF estimation problem lies primarily in obtaining realistic values for losses.

$$\text{CARF} = \left[\frac{\text{Losses: the number of items that will require replacement during a 30-day period}}{\text{Number of items in use}}\right]\left[100\right]$$

Failure to place correct values here can lead to CARFs which are substantially in error, even though losses are well estimated.

FIGURE 2. The Anatomy of a CARF

its most frequent application will be that of multiplying a quantity of equipment to establish the quantity that will need replacement. Accordingly, it is crucial if errors are to be avoided that the quantity the CARF multiplies be of the same substance as the denominator used to determine the CARF value.

There are several ways to interpret in-use amount, and as suggested above, clarification might usefully be based upon CARF application. This appears to have been done as follows, quoting from the War Reserve Policy Manual.[2]

> 1. "The combat active replacement factor will be applied for units during those periods when they are actually in active combat operations. A force in contact with the enemy is considered to be in active combat." (pg 4-3,4-4)
>
> 2. "When applied to the density of equipment, the combat active replacement factor determines the amount of equipment which must be replaced each month to maintain the full T/E allowance."(pg 4-4)
>
> 3. " . . . (CARFs) multiplied by the equipment density or unit Table of Equipment (T/E) allowance." (pg 12-5)

As has been said, as long as the denominator used in specifying the CARF and the quantity the CARF multiplies are of the same nature, there will be no difficulty. However, we will have errors that may be substantial when a CARF is used to multiply, say, the number of items in the FMF.

Another area where there may be difficulty with the CARF denominator occurs when a CARF is based upon an Army replacement factor, called a WARF. The Army's

> "Wartime Replacement Factor (WARF) is the average daily catastrophic (nonrepairable) item loss rate expressed as a percent of the average authorized item strength in the combat theater".

To let CARF=(WARF)(30) means (assuming all other aspects of Army and Marine operations to be similar) that "average authorized item strength in the combat theater" is the same concept as the Marine's "in-use amount".

Percentage or Proportion?

Current numerical values for CARFs for various items are listed in the Table of Authorized Materiel (TAM).[3] Consistent with definitions given elsewhere, the TAM defines replacement factors as percentages and then lists the values for "replacement factors for 30 days" beginning on page 26-1. However, the values that are listed are in fact proportions, 100 times smaller than the replacement factors described. It is apparently true that Supply Officers have been trained to interpret these values as proportions. However, others seeking CARF values will probably go to the TAM as the basic and most available source. The TAM says clearly that they are percentages, and thus for unknowing users, the values they extract and use are actually understating losses by a factor of 100.

APPENDIX B

DEVELOPMENT OF THE RANKING METHOD

The procedures in Section II of this report employed a ranking method by which experts (called judges) expressed their feelings about loss rates or CARF values (scale values or amount of the parameter) associated with various items of equipment (instances)) by ranking the items. The collective rankings of the judges were then used to produce interval scaled values for the amount of the property possessed by each of the instances.

There are several ways to approach scale development from ordinal data. Models vary, depending upon what assumptions are made. We shall present what is probably the most widely used approach.[11] We assume the following:

a. A judge cannot directly express his feelings x_j about the scale value of instance j, but is able to rank instances in accordance with his feelings.

b. Over the population of judges, x_j is a normally distributed random variable.

c. All instances possess the same variance* for x, so that $\sigma_j^2 = \sigma^2$.

d. The correlation coefficient for x between any pair of instances is the same, so that $\rho_{ij} = \rho$.

From these assumptions we may deduce the following. Let i and j be tw instances. A judge's feeling about the amount of the property possess by instance i is a normally distributed random variable x_i with mean S_i and variance σ^2, and a judge's feelings about the amount of the pr possessed by instance j is normally distributed random variable x_j wit mean S_j and variance σ^2. Since the difference between two normally distributed random variables is itself a normally distributed random variable, $(x_i - x_j)$ is normal with mean $S_i - S_j$ and variance $\sigma^2 + \sigma^2 - 2\rho\sigma^2 = 2\sigma^2(1-\rho)$ where ρ is the correlation coefficient. Now, the probability that a judge rates instance j as possessing more of the property than instance i may be expressed as $Pr(x_j > x_i)$. We will operate on the inequality as follows:

$$Pr(x_j > x_i) = Pr\ (0 > x_i - x_j)$$

$$= Pr\left[-(S_i - S_j) > (x_i - x_j) - (S_i - S_j)\right]$$

$$= Pr\left[\frac{S_j - S_i}{\sqrt{2\sigma^2(1-\rho)}} > \frac{(x_i - x_j) - (S_i - S_j)}{\sqrt{2\sigma^2(1 - \rho)}}\right] .$$

The right hand side of the final inequality above is a normally distr buted random variable with a mean of zero and a variance of unity. Th we have

$$Pr(x_j > x_i) = Pr\left(\frac{S_j - S_i}{\sqrt{2\sigma^2(1 - \rho)}} > z \right), \tag{16}$$

where z is the standard normal deviate.

An estimate of the left-hand side of (16) may be obtained from the ranking information furnished by the judges. The proportion of judges who rank instance j as possessing more of the property than instance i may be used as an estimate of $Pr(x_j > x_i)$.

Let p_{ij} be the proportion of judges who rate instance j as possessing more of the property to be scaled than instance i. Let z_{ij} be the value of the standard normal deviate (from the Normal Table) associated with p_{ij}, that is, z_{ij} is the value of z for which the leftward area under the normal N(z;0,1) curve is p_{ij}.

Examples are

p_{ij}	z_{ij}
0.92	1.645
0.90	1.282
0.50	0
0.001	-3.0

We now have, from Equation (16) estimating equations of the form

$$z_{ij} = \frac{S_j - S_i}{\sqrt{2\sigma^2(1 - \rho)}}, \tag{17}$$

with one of these equations occurring for each instance pair i,j. In (17), the left hand z_{ij} values come from the judges' rankings, being the standard normal deviate associated with the proportion of judges who ranked instance j as possessing more of the property than instance i. On the right hand side of (17), we have $S_j - S_i$, the difference in two of the scale values we wish to obtain.

Since we seek values of S_i and S_j on an interval scale whose uni[t] and origin are yet unspecified, we may use our freedom to specify unit and origin to simplify the mathematical development. Reserving specification of the scale's origin until later, we can currently obtain a simpler form of (17) by specifying a unit for the scale such that

$$\sqrt{2\sigma^2(1-\rho)} = 1. \tag{18}$$

This is, of course, not an assumption.

The scaling problem now stands as follows. We have n instances to be scaled; we seek values for S_1, S_2, ..., S_n. We have an n x n array of z_{ij} values which came from judges' rankings, and we have a set of n(n-1)/2 estimates of the form

$$z_{ij} \approx S_j - S_i \ . \tag{19}$$

For n > 3, we will have more estimates than values to be estimat[ed]. Viewing the various estimates as being equally useful, we will now propose a least-squares fit over the estimates (19).

Before going ahead with the least-squares procedure, we must at this time pause to point out an important characteristic of the z_{ij} arr[ay]. If all judges rank, say, instance a as posessing more of the property than instance b, then we will have $p_{ba} = 1.0$, $p_{ab} = 0$, and thus $z_{ba} = \infty$ and $z_{ab} = -\infty$. In practice, to avoid numerical bias by a small number of judges, z_{ij} values corresponding to

$$p_{ij} > 0.98$$

and

$$p_{ij} < 0.02$$

are omitted from the z_{ij} array.[14] Thus (if any) there will be an even number of "holes" in the z_{ij} array, symmetric about the diagonal. Our continued development must take into account the possibility that the z_{ij} array may be incomplete.

Least-Squares Solution

For the least-squares procedure, let ϕ_j be the set of rows possessing elements in the jth column of the z_{ij} array. Now ideally, $z_{ij} = S_j - S_i$, and thus for our least-squares fit, we will want to find a value for S_j that minimizes

$$W_j = \sum_{i\varepsilon\phi_j} \left(z_{ij} - (S_j - S_i)\right)^2 .$$

The algebraic expansion of the summation term is

$$W_j = \sum_{i\varepsilon\phi_j} \left[z_{ij}^2 - 2z_{ij}(S_j - S_i) + (S_j - S_i)^2 \right]$$

$$= \sum_{i\varepsilon\phi_j} \left[z_{ij}^2 - 2z_{ij}S_j + 2z_{ij}S_i + S_j^2 - 2S_jS_i + S_i^2 \right]$$

$$= \sum_{i\varepsilon\phi_j} z_{ij}^2 - 2S_j \sum_{i\varepsilon\phi_j} z_{ij} + 2\sum_{i\varepsilon\phi_j} z_{ij}S_i + n_jS_j^2 - 2S_j \sum_{i\varepsilon\phi_j} S_i + \sum_{i\varepsilon\phi_j} S_i^2 ,$$

where n_j is the number of elements of ϕ_j. For a minimum, we take the derivative and equate it to zero, viz.,

$$\frac{\partial W_j}{\partial S_j} = -2\sum_{i\varepsilon\phi_j} z_{ij} + 2n_jS_j - 2\sum_{i\varepsilon\phi_j} S_i \overset{set}{=} 0,$$

or

$$n_jS_j - \sum_{i\varepsilon\phi_j} S_i = \sum_{i\varepsilon\phi_j} z_{ij} \quad , j = 1,2,\ldots,n. \tag{20}$$

Equation (20) represents a set of n linear equations in S_1, S_2, ..., S_n. It may be shown that this set of equations is not independent (the read may wish to establish this by showing that the negative of any equation is equal to the sum of all remaining equations, or that the coefficient matrix is singular). The set of equations becomes independent when any one equation is eliminated, leaving us with a set of n-1 equations in n unknowns. After arbitrarily setting a value for one of the S_j (which i equivalent to setting the origin for the interval scale we are developi we can solve the set of equations using standard procedures such as Gauss-Jordan elimination methods.[17]

Solving sets of simultaneous equations becomes a cumbersome activity when appropriate computing equipment is unavailable and when n is even modestly large. This is true even when the coefficient matr consists of small integers, as is our case. A method is given below t reduce this effort by decreasing the number of equations which have to be fully solved. Savings in effort will occur only when there are col in the z_{ij} array which are complete in that they have no missing elements.

To present our labor-saving approach, we return to our set of n equations of the form

$$n_j S_j - \sum_{i \epsilon \phi_j} S_i = \sum_{i \epsilon \phi_j} z_{ij} \; ; \; j = 1, 2, \ldots, n. \tag{20}$$

If there exists one or more complete columns in the z_{ij} array, we procede by establishing the origin for the scale by making the average scale value zero, i.e.,

$$\frac{\sum_{i=1}^{n} S_i}{n} = 0 \quad , \tag{21}$$

which implies that $\sum_{i=1}^{n} S_i = 0$.

Suppose that column k is complete, so that $n_k = n$. The equation for column k is

$$n\, S_k - \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} z_{ik} \quad ,$$

and immediately,

$$S_k = \frac{\sum_{i=1}^{n} z_{ik}}{n} \quad , \tag{22}$$

since $\sum_{i=1}^{n} S_i = 0$. Thus, when a column is complete, we can immediately compute the scale value for the instance associated with that column as the average of the z_{ij} in that column. After we have done this for all complete columns, there will remain a set of incomplete columns associated with instances for which we do not yet have the scale values. All that remains is to (a) write out the equations of the form of (20) for the incomplete columns, substitute the values of the s_j we have obtaine from the complete columns, and (b) then solve this smaller, easier set of equations.

In the procedure just described, column averages of complete z_{ij} columns are used as scale values. Thus if all columns are complete, th entire set of scale values S_j may be readily found by simply using colu averages. It must be emphasized that when elements are missing from a column, it is not correct to use that column average as an estimate of scale value, even for an approximation.

APPENDIX C.

A NUMERICAL EXAMPLE

Two hundred judges were asked to rank four types of equipment in terms of loss rate. They were instructed to record the item (A, B, C, and D) in rank order according to their best judgement, but to omit ranking any type of equipment with which they felt too unfamiliar to make a judgement about its loss rate. They were also instructed that if they felt that two items had the same loss rate, they should be recorded on the same line.

A typical judge's response, then, would be a completed from as shown below.*

Highest Loss Rate	B
	A
	C
Lowest Loss Rate	D

This input was tallied on the frequency array in the following manner. Since B was rated at the top of this judge's ranking, we go to the B column of the frequency array and in that column we record tallies in rows corresponding to instances (items) that this judge has ranked

*Note the cues given to the first and last rankings, which help reduce the chance that a judge will list instances in reverse order.

below B.:

f_{ij}	A	B	C	D
A		1		
B				
C		1		
D		1		

Then we move to the judge's second choice, find its column, and reco
tallies in rows corresponding to instances ranked below the second ch

f_{ij}	A	B	C	D
A		1		
B				
C	1	1		
D	1	1		

Continuing to the judge's third ranked instance, C we obtain

f_{ij}	A	B	C	D
A		1		
B				
C	1	1		
D	1	1	1	

as this judge's contribution to the total tally for all judges.

The completed frequency array for all 200 judges is shown below.

f_{ij}	A	B	C	D
A		33	1	45
B	167		30	118
C	195	166		172
D	155	82	24	

Adding across the diagonal, we see that not all 200 judges made all comparisons. This was due to the fact that several judges omitted item C from their rankings.

Continuing, we next must compute the p_{ij} array from the f_{ij} array by

$$p_{ij} = \frac{f_{ij}}{f_{ij} + f_{ji}} .$$

Because not all judges ranked all items, the divisor in some cases will be less than 200. The p_{ij} array is computed as

p_{ij}	A	B	C	D
A	0.5	0.165	0.005	0.225
B	0.835	0.5	0.153	0.590
C	0.995	0.847	0.5	0.878
D	0.775	0.410	0.122	0.5
sums	2.605	1.420	0.280	1.693

From the p_{ij} array we see that in comparing C and A the judges were too close to unanimity, and thus we will omit this comparison ($p_{AC} < 0.02$, $p_{CA} > 0.98$) in the z_{ij} array. From a table of the normal distribution, the z_{ij} array is

z_{ij}	A	B	C	D
A	0	-0.974		-0.755
B	0.974	0	-1.024	0.227
C		1.024	0	1.170
D	0.755	-0.227	-1.170	0

.

Since the z_{ij} array has cells which are empty, we cannot use t column averages to compute scale values for all the items. Instead, w have to solve simultaneous equations to obtain a least-square solution. Two columns of the z_{ij} array are complete, and two are not. We begin by computing the column sums:

j	A	B	C	D
$\sum\limits_{i\varepsilon\phi j} z_{ij}$	1.729	-0.177	-2.194	0.642
n_j	3	4	3	4

.

Implicitly setting the origin of the scale by taking the average scale value as zero, we immediately compute column averages for the two complete columns and obtain two scale values:

$$S_B = \frac{-0.177}{4} = -0.044$$

and

$$S_D = \frac{0.642}{4} = 0.161 \quad .$$

For the remaining columns (A and C), the set of equations of t form of (5) is

$$3S_A - (S_A + S_B + S_D) = 1.729$$

$$3S_C - (S_B + S_C + S_D) = -2.194.$$

Since

$$S_B + S_D = 0.117 ,$$

we may write the two equations as

$$2S_A = 1.729 + 0.117$$

$$= 1.846$$

and

$$2S_C = -2.194 + 0.117$$

$$= -2.077 ,$$

and thus we have

$$S_A = 0.923$$

and

$$S_C = -1.039 .$$

These values are still with unspecified unit and origin. If we estimate that the loss rate for C is 10, and the loss rate for item D is 30, then Equation 15 becomes

$$L_j = 27.32 + 16.67S_j ,$$

and we have, finally,

Item	Loss Rate
A	42.7
B	25.6
C	10.0
D	30.0

APPENDIX D

RANKINGS BY JUDGES

Judge:	1	2	3	4	5	6	7	8	9	10
Highest Chance of Loss	46	40	40	43	41		13	13	20	46
	41	41	41	10	40		14	14	23	40
	42	42	42	16	42	40	49	49	26	41
	40	43	19	20	43	41	43	40	29	42
	10	49	14	19	13	42	41	42	23	13
	16	13	13	14	30	30	40	43		49
	43	14	30	17	35	46	42	41		43
	49	16	10	46	49	10		17		14
	13	30	26	42	46	20	46	11		17
	30	17	43	41	29	43	30	16		19
	35	23	49	40	23	29	19	30		30
	14	35	16		10	16	10	35		23
	17	29	17		17	17	29	10		26
	23	26	20	43	14	19	23	19		35
	26	20	21	49	19	14	50	23		16
	19	19	29	30	20	13	26	20		20
	20	46	23	35	26	21	11	26		21
	29	21	35	29	21	23	20	21		
	21	10		26		26	35	29		
	50			23		35	16	46		
Lowest Chance of Loss				21		49	21	50		
Delete:				43					23	
Omits:	11	11 50	11 46 50	11 13 50	11 16 50	11 50	17		many	11 40

Judge:	11	12	13	14	15	16	17	18
Highest Chance of Loss	41	41	41	13	14	42	16,10	40,43
	40	40	11	14	17	41	41	42
	42	42	10	49	13	43	42	30
	23	23	16	43	41	14	49	46,49
	30	30	40	40	42	10	40	13
	35	35	42	17	30	13	43	14
	29	29	43	42	40	30	13	16
	19	19	30	19	43	35	14	20
	10	10	35	10	20	40	17	10
	11	11	26	16	35	17	30	19
	26	26	49	46	10	29	35	29
	13	13	23	11	29	46	19	
	14	14	46	30	26	49	23	
	16	16	20	29	21	13	29	
	17	17	29	35	49	20	26	
	20	20	17	23	46	19	20	
	21	21	19	26	16	21	21	35
			13	20	23	23	46	
	46	46	14	21	19	26		
	43	43	21	50	11	50		
Lowest Chance of Loss	49	49	50		50			26,21
Delete:						13		
Omits:	50	50		41		11 16	11 50	11 17 23 41 50

Judge:	19	20	21	22
Highest Chance of Loss	13,14	13	13	49
	43,49	49,17,14	43	13,14
	17	40	49	17
	16,19	41	14	20,19,16,21
	40,41,42	10,11	40,41,42	40
	29,30	43	10	43
	10,11	16	16	42
	20,26	42	17	41
	21	30,19,35		29
	23	26,23,29	30	10,23
	46	20,46	35	26,46
	35	21		30,35
	50		29	
			26	
			19	
Lowest Chance of Loss			20,21,23	
Delete:				
Omits:		50	11 46 50	11 50

Judge:	23
Highest Chance of Loss	13,19
	49
	40,41,42,43
	30,35
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